Other Use of Triangle Inequality

Algorithms for Nearest Neighbor Search: Lecture 2

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Outline

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1. Nearest Neighbors via Walking
2. Matrix-Based Techniques

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2. Matrix-Based Techniques
3. Basic Techniques for Euclidean Space

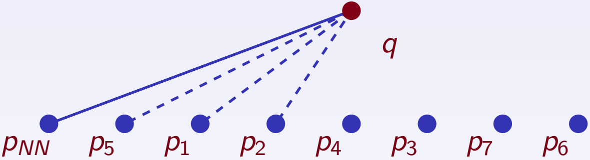
Chapter VI

Nearest Neighbors via Walking

Orchard's Algorithm

Preprocessing: Orchard'91

For every object $p_i \in S$ construct a list $L(p_i)$ of all other objects sorted by their similarity to p_i



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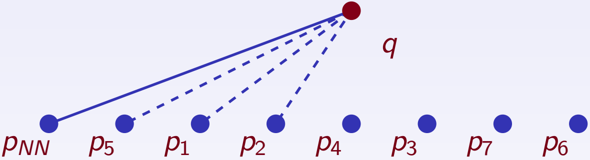

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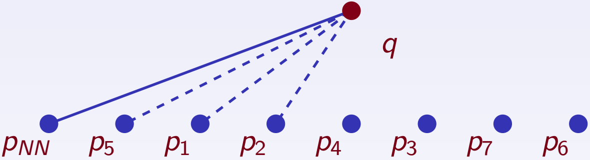

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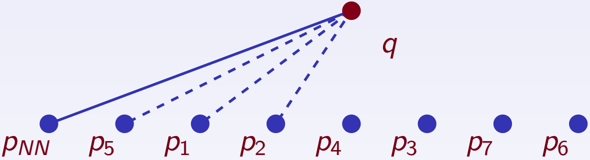

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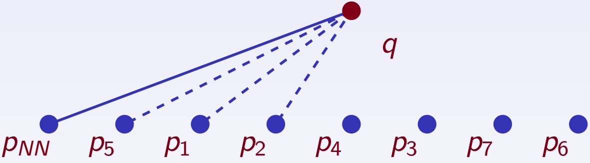

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- Stopping condition: we reached p' having $d(p', q) \geq 2d(p_{NN}, q)$

Hierarchical Orchard's Algorithm

- Randomly choose $S_1 \subset S_2 \subset \ldots S_k = S$ with $|S_i|/|S_{i-1}| \approx \alpha > 1$
- Start with Orchard algorithm on S_1
- For every i from 2 to k apply Orchard's algorithm for S_i using result of the previous step as a starting point

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Inspired by classic skip list technique   Pugh'90

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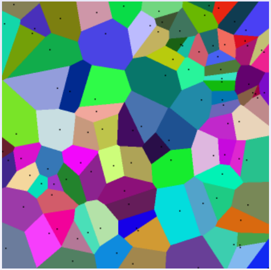
Delaunay Graph:

Construct Voronoi diagram for set in Euclidean space. Draw an edge between every two points whose Voronoi cells are adjacent



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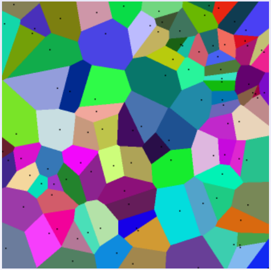

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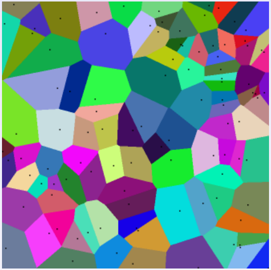

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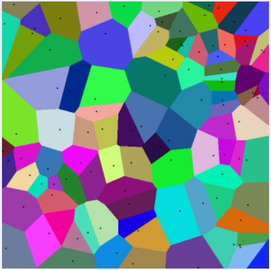

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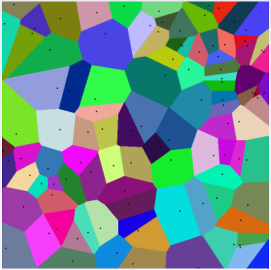

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- Otherwise return p

Delaunay Graph in General

Exercise: prove correctness of the above algorithm

Assume we have general metric space and full matrix of pairwise distances. How Delaunay graph should be defined?

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Navarro, 2002: for any distance matrix any two objects can be adjacent :-(

Spatial Approximation Tree: Construction

Navarro'99:

- Set a random object p to be root
- Partitioning technique:
  - Inspect all other object in order by their similarity to p
  - Whenever some p' is closer to p than to any of already chosen children $Ch(p)$ add p' to children set
  - Put every other object p'' to the subtree of closet member of $Ch(p)$
- Recursively repeat

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Exercise: prove that covering radius for children subtree is never exceeding covering radius of parent subtree

SA-Tree: Search

- Start from the root p
- For every node to be inspected:

  keep global candidate p_{NN}

  (closest object to query visited so far)

  and p_a — closest to q among

  all ancestors and brothers of current node
- Use usual depth-first or best-first tree traversal
- Processing current node t:
  - Compute distances from q to all children of t
  - Go to child s whenever $d(q, s) < d(q, p_a(s)) + 2r_{NN}$

SA-Tree: Correctness

Observation: fix node s, let p_a be its ancestor/brother and s' be some objected in its subtree. Then s' is closer to s than to p_a

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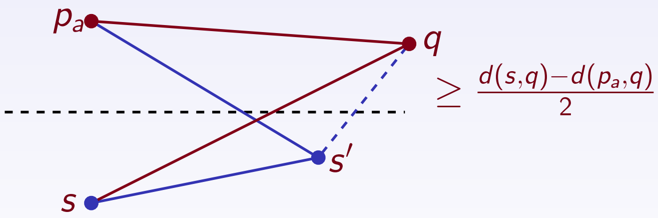

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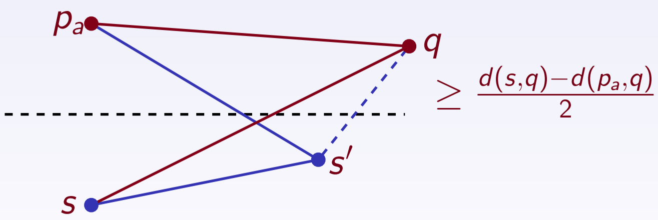

If there exists s' such that $d(s', q) < r_{NN}$ then $d(s, q) < d(p_a, q) + 2r_{NN}$

Chapter VII

Matrix-Based Techniques

Approximating and Eliminating Search Algorithm

Preprocessing: Vidal'86

Compute $n \times n$ matrix of pairwise distances in S

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Query processing:

- Maintain a set C of candidate objects, initially $C := S$
- For every $p \in C$ keep the lower bound $d_l(q, p)$
- Main loop:
  - Choose $p \in C$ with smallest lower bound, compute $d(q, p)$, update $p_{NN}, r_{NN} = d(q, p_{NN})$ if necessary
  - Approximating: update lower bounds in C using $d(q, p') \geq d(q, p) + d(p, p')$ inequality
  - Eliminating: delete all elements in C whose lower bounds exceeded r_{NN}

Linear AESA

Advantage of AESA: small number of distance computations
Disadvantages: large storage and non-distance computation

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Linear AESA: Micó, Oncina, Vidal'94

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Compute $n \times m$ matrix choosing m objects as pivots

Range search:

- Compute all query-pivot distances
- Compute lower bounds for all non-pivot objects
- Eliminate objects with lower bound exceeding search range
- Explicitly check remaining non-pivots

TLAESA

A combination of bisector tree and LAESA

Data structure: Micó, Oncina, Carrasco'96

Usual bisector tree

Additionally, m pivots

Distances from pivots to all objects are precomputed

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Query processing

Compute distances from query to pivots

Depth-first/Best-first search in bisector tree

Additional condition to prune subtree of some object s:

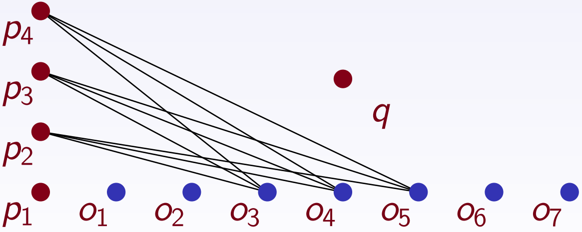
$$\exists i : \quad |d(p_i, s) - d(p_i, q)| \geq r_c(s) + r_{NN}$$

Shapiro's Algorithm (1/2)

Data structure: Shapiro'77

$n \times m$ distance matrix (pivots $p_1, \ldots, p_m$)

Non-pivot objects are sorted by there distances to first pivot $p_1 : o_1, \ldots, o_n$



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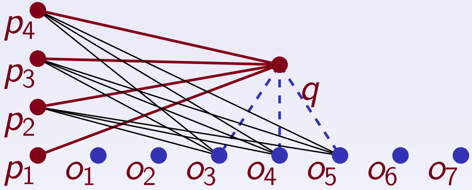

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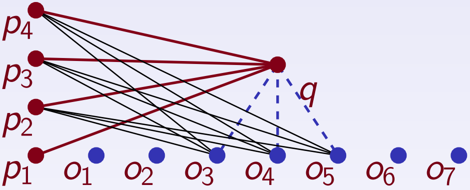

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Start with o_i having $d(p_1, o_i) \approx d(p_1, q)$

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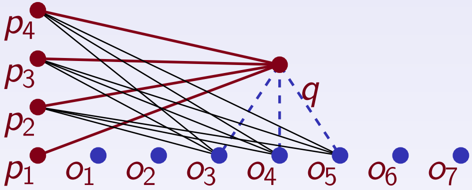

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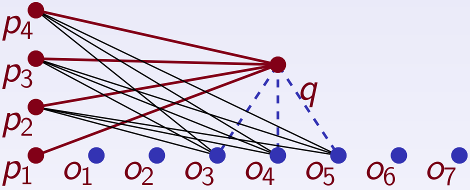

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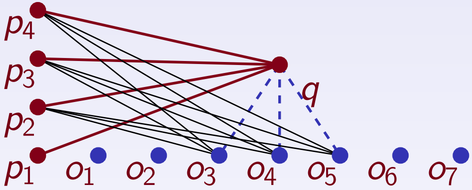

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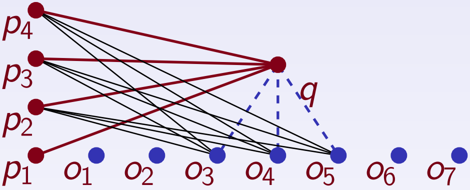

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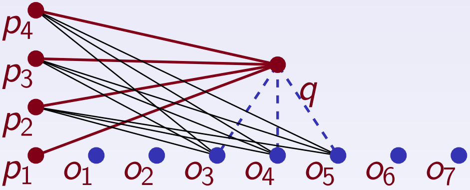

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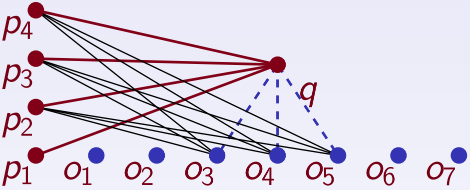

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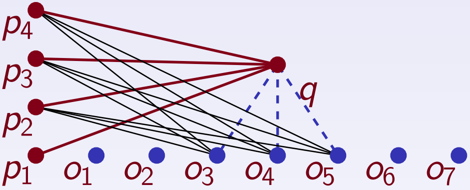

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But published before both: 1977 vs 1991 and 1992!

Chapter VIII

Basic Techniques for Euclidean Space

Advantages of Euclidean Space

- Rich mathematical formalisms for defining a boundary of any set

  Examples: rectangles, hyperplanes, polynomial curves

- Easy computation of lower bound on distance between query point and any set boundary

- (Tomorrow) Easy definable mappings to smaller spaces

k-d Tree

Preprocessing: Bentley, 1975

Top-down partitioning

On level l: split the current set

by hyperplane orthogonal to $l \bmod k$ axis

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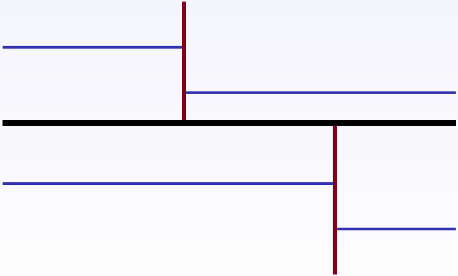

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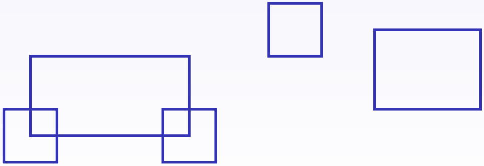

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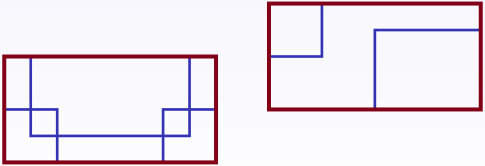

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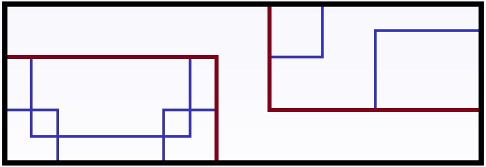

Exercises

Prove correctness of Delaunay graph stopping condition

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Prove monotonicity of covering radii in SA-tree

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Thanks for your attention! Questions?

References

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